

Two-Dimensional Motion and Vectors

Problem D**PROJECTILES LAUNCHED HORIZONTALLY****PROBLEM**

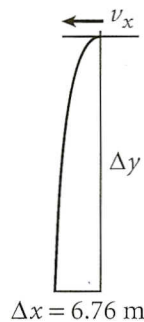
Although not the fastest or tallest or steepest roller coaster in the world, the “High Roller” roller coaster atop the Stratosphere Tower, in Las Vegas, Nevada, is the highest. Suppose that during construction of the ride a metal bolt was accidentally knocked horizontally off the edge of the Stratosphere. If the bolt’s initial speed was 0.80 m/s, it would have traveled 6.76 m in the horizontal direction before hitting the ground. Use this information to calculate how tall the Stratosphere Tower is.

SOLUTION**1. DEFINE**

Given: $v_x = 0.80 \text{ m/s}$
 $\Delta x = 6.76 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

Unknown: $\Delta y = ?$

Diagram:

**2. PLAN**

Choose the equation(s) or situation: The magnitude of the vertical displacement is given by the equation for falling bodies with no initial vertical velocity.

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2$$

The magnitude for horizontal displacement is given by the equation for displacement at constant velocity.

$$\Delta x = v_x \Delta t$$

Rearrange the equation(s) to isolate the unknown(s): Substitute for Δt in the falling-body equation.

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2} a_y \left(\frac{\Delta x}{v_x} \right)^2$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\Delta y = \frac{1}{2} (-9.81 \text{ m/s}^2) \left(\frac{6.76 \text{ m}}{0.80 \text{ m/s}} \right)^2$$

$$= -350 \text{ m}$$

height of building = 350 m

4. EVALUATE

The solution can be checked by using both equations to solve for Δt . From the equation for falling bodies, Δt is found to be 8.4 s. From the equation for horizontal displacement, Δt is 8.4 s. Both times are the same, so Δy is correctly calculated.

ADDITIONAL PRACTICE

1. Lookout Mountain, which overlooks the Tennessee River Valley near Chattanooga, Tennessee, was of great strategic importance during the Civil War. Today, some of the artillery used in the war remain at the park located on top of the mountain. Suppose one of these cannons fired a projectile horizontally with a speed of 430 m/s, so that the projectile landed at a horizontal distance of 4020 m from the cannon. How high would the ridge of the mountain be with respect to the valley below?
2. In 1977, a helicopter at the heliport atop the 59-story Pan Am building in New York fell over, causing the rapidly-turning rotor blades to splinter. One of these fragments landed about 101 m away, near the corner of Madison Avenue and 43rd Street. Suppose the fragment moved off the building horizontally with a speed of 14.25 m/s. Use this information to find the height of the Pan Am building.
3. The LZ N07 is a newly designed airship in the manner of the old Zeppelin airships built in Germany between 1908 and 1940. New technology has made the LZ N07 more efficient and safe, as well as speedier. This airship can travel with a horizontal speed of up to 1.30×10^2 km/h. If a parcel is dropped from this airship, so that it lands 135 m in front of the spot over which it was released, how far above the ground is the airship?
4. The shape of Sugarloaf mountain, in Rio de Janeiro, Brazil, is such that, if you were to kick a soccer ball hard enough, it could land near the base of the mountain without hitting the mountain's side. Suppose the ball is kicked horizontally with an initial speed of 9.37 m/s. If the ball travels a horizontal distance of 85.0 m, how tall is the mountain?
5. Although many structures taller than 500 m have been designed, few have been built due to practical limitations, such as cost and safety. In light of this, the Bionic Tower in Hong Kong may never be more than a design. If it is built, the Bionic Tower will provide working space for 100,000 people, and transport them using over 300 elevators. Suppose a plate-glass window falls out of place from the top floor of the Bionic Tower. Although the window's speed is only 6.32 cm/s in the horizontal direction, the window will still have a horizontal displacement of 1.00 m once it hits the street below. Use this information to calculate the proposed height of the Bionic Tower.
6. A squirrel on a limb near the top of a tree loses its grip on a nut, so that the nut slips away horizontally at a speed of 10.0 cm/s. If the nut lands at a horizontal distance of 18.6 cm, how high above the ground is the squirrel?

7. A lunch pail is accidentally kicked off a steel beam on a building under construction. Suppose the initial horizontal speed is 1.50 m/s. How far does the lunch pail fall after it travels 3.50 m horizontally?
8. If the building in problem 7 is 2.50×10^2 m tall, and the lunch pail is knocked off the top floor, what will be the horizontal displacement of the lunch pail when it reaches the ground?
9. What is the velocity of the lunch pail in problem 8 when it reaches the ground?
10. What is the range of an arrow shot horizontally at 85.3 m/s if it is initially 1.50 m above the ground?

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1) $v = 430 \text{ m/s}$
 $D_x = 4020 \text{ m}$
 $a = -9.81 \text{ m/s}^2$
 $D_y = ?$

$$D_x = vt$$
$$\frac{4020}{430} = \frac{430}{430} t$$
$$9.349 = t$$

$$D_y = \frac{1}{2} at^2$$

$$D_y = \frac{1}{2} (-9.81) (9.349)^2$$

$$D_y = -437.01 \text{ m}$$

2) $D_x = 101 \text{ m}$

$v = 14.25 \text{ m/s}$
 $a = -9.81 \text{ m/s}^2$
 $D_y = ?$

$$D_x = vt$$
$$\frac{101}{14.25} = \frac{14.25}{14.25} t$$

$$7.088 \text{ sec} = t$$

$$D_y = \frac{1}{2} at^2$$

$$D_y = \frac{1}{2} (-9.81) (7.088)^2$$

$$D_y = -246.426 \text{ m}$$

3) $v = 3.611 \text{ m/s}$

$D_x = 135 \text{ m}$
 $a = -9.81 \text{ m/s}^2$
 $D_y = ?$

$$D_x = vt$$
$$\frac{135}{3.611} = t$$

$$37.386 = t$$

$$D_y = \frac{1}{2} at^2$$

$$D_y = \frac{1}{2} (-9.81) (37.386)^2$$

$$D_y = -6855.782 \text{ m}$$

4. $V_i = 9.37 \text{ m/s}$
 $D_x = 85.0 \text{ m}$
 $a = -9.81 \text{ m/s}^2$

$D_x = Vt$
 $\frac{85}{9.37} = t$

$t = 9.07 \text{ sec}$

$D_y = \frac{1}{2} a t^2$
 $D_y = \frac{1}{2} (-9.81)(9.07)^2$
 $D_y = 403.51 \text{ m}$

5. $V = 0.632 \text{ m/s}$
 $D_x = 1 \text{ m}$

$t = (0.632) t$
 $t = 1.582$

$D_y = \frac{1}{2} (-9.81)(1.582)^2$

$D_y = 1227.586 \text{ m}$

6. $V = 0.1 \text{ m/s}$
 $D_x = 0.186 \text{ m}$
 $D_x = VT$
 $\frac{0.186}{0.1} = t$
 $t = 1.86 \text{ sec}$

$D_y = \frac{1}{2} (-9.81)(1.86)^2$

$D_y = 16.969 \text{ m}$

7. H
 $d = 3.5 \text{ m}$
 $v = 1.5 \text{ m/s}$

V
 $D = ?$
 $A = -9.81 \text{ m/s}^2$
 $T = 2.3 \text{ s}$

$\frac{3.5}{1.5} = 2.3$

$d = \frac{1}{2} (-9.81)(2.3)^2 = 26.705 \text{ m}$

8. H
 $v = 1.5 \text{ m/s}$
 $t = 7.14 \text{ s}$
 $d = ?$

V
 $D = 250 \text{ m}$
 $A = -9.81$

$t = \sqrt{\frac{250}{(-9.81)}} = 7.14 \text{ s}$

$1.5 \times 7.14 = 10.71 \text{ m}$

9. $t \cdot a = V_f$

$V_f = -70.0434 \text{ m/s}$

$\frac{V_f - V_i}{t} = a$
 $\frac{-70.0434 - 1.5}{7.14} = -9.81$

10. $D_y = 1.5\text{m}$
 $v = 85.3\text{m/s}$
 $a = -9.81\text{m/s}^2$

$D_x = Vt$
 $85.31(.553)$
 $D_x = 47.171\text{m}$

$D_y = \frac{1}{2} at^2$
 $t = .553$