Two-Dimensional Motion and Vectors

Problem F

RELATIVE VELOCITY

PROBLEM

The world's fastest current is in Slingsby Channel, Canada, where the speed of the water reaches 30.0 km/h. Suppose a motorboat crosses the channel perpendicular to the bank at a speed of 18.0 km/h relative to the bank. Find the velocity of the motorboat relative to the water.

SOLUTION

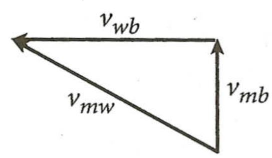
1. DEFINE

Given: $\mathbf{v_{wb}}$ = 30.0 km/h along the channel (velocity of the *water, w,* with respect to the *bank, b)*

$\mathbf{v_{mb}}$ = 18.0 km/h perpendicular to the channel (velocity of the *motorboat, m,* with respect to the *bank, b)*

Unknown: $\mathbf{v_{mw}} = ?$

Diagram:



2. PLAN

Choose the equation(s) or situation: From the vector diagram, the resultant vector (the velocity of the motorboat with respect to the bank, $\mathbf{v_{mb}}$) is equal to the vector sum of the other two vectors, one of which is the unknown.

$$\mathbf{v_{mw}} = \mathbf{v_{mb}} + \mathbf{v_{wb}}$$

Use the Pythagorean theorem to calculate the magnitude of the resultant velocity, and use the tangent function to find the direction. Note that because the vectors $\mathbf{v_{mb}}$ and $\mathbf{v_{wb}}$ are perpendicular to each other, the product that results from multiplying one by the other is zero. The tangent of the angle between $\mathbf{v_{mb}}$ and $\mathbf{v_{mw}}$ is equal to the ratio of the magnitude of $\mathbf{v_{wb}}$ to the magnitude of $\mathbf{v_{mb}}$.

$$v_{mw}^2 = v_{mb}^2 + v_{wb}^2$$

$$\tan\theta = \frac{v_{wb}}{v_{mb}}$$

Rearrange the equation(s) to isolate the unknown(s):

$$v_{mw} = \sqrt{v_{mb}^2 + v_{wb}^2}$$

$$\theta = \tan^{-1}\left(\frac{v_{wb}}{v_{mb}}\right)$$

3. CALCULATE

Substitute the values into the equation(s) and solve: Choose the positive root for v_{mw}.

$$v_{mw} = \sqrt{\left(18.0\,\frac{\text{km}}{\text{h}}\right)^2 + \left(30.0\,\frac{\text{km}}{\text{h}}\right)^2} = \boxed{35.0\,\frac{\text{km}}{\text{h}}}$$

The angle between v_{mb} and v_{mw} is as follows:

$$\theta = \tan^{-1}\left(\frac{30.0\,\frac{\text{km}}{\text{h}}}{18.0\,\frac{\text{km}}{\text{h}}}\right) = \boxed{59.0^\circ \text{ away from the oncoming current}}$$

4. EVALUATE The motorboat must move in a direction 59° with respect to $\mathbf{v_{mb}}$ and against the current, and with a speed of 35.0 km/h in order to move 18.0 km/h perpendicular to the bank.

ADDITIONAL PRACTICE

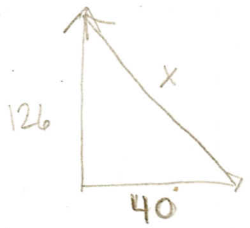
1. In 1933, a storm occurring in the Pacific Ocean moved with speeds reaching a maximum of 126 km/h. Suppose a storm is moving north at this speed. If a gull flies east through the storm with a speed of 40.0 km/h relative to the air, what is the velocity of the gull relative to Earth?
2. George V Coast in Antarctica is the windiest place on Earth. Wind speeds there can reach 3.00×10^2 km/h. If a research plane flies against the wind with a speed of 4.50×10^2 km/h relative to the wind, how long does it take the plane to fly between two research stations that are 250 km apart?
3. Turtles are fairly slow on the ground, but they are very good swimmers, as indicated by the reported speed of 9.0 m/s for the leatherback turtle. Suppose a leatherback turtle swims across a river at 9.0 m/s relative to the water. If the current in the river is 3.0 m/s and it moves at a right angle to the turtle's motion, what is the turtle's displacement with respect to the river's bank after 1.0 min?
4. California sea lions can swim as fast as 40.0 km/h. Suppose a sea lion begins to chase a fish at this speed when the fish is 60.0 m away. The fish, of course, does not wait, and swims away at a speed 16.0 km/h. How long would it take the sea lion to catch the fish?
5. The spur-wing goose is one of the fastest birds in the world when it comes to *level* flying: it can reach a speed of 90.0 km/h. Suppose two spur-wing geese are separated by an unknown distance and start flying toward each other at their maximum speeds. The geese pass each other 40.0 s later. Calculate the initial distance between the geese.
6. The fastest snake on Earth is the black mamba, which can move over a short distance at 18.0 km/h. Suppose a mamba moves at this speed toward a rat sitting 12.0 m away. The rat immediately begins to run away at 33.3 percent of the mamba's speed. If the rat jumps into a hole just before the mamba can catch it, determine the length of time that the chase lasts.

Problem F - Relative Velocity

1) $V_{SE} = 126$ km/hr N y

$V_{GS} = 40$ km/hr E x

$V_{GE} = ?$



72.4 N of E

$V_{GE} = V_{GS} + V_{SE}$

$V_{GE} = 40 + 126$ $\sqrt{40^2 + 126^2} =$ 132.2 km/hr.

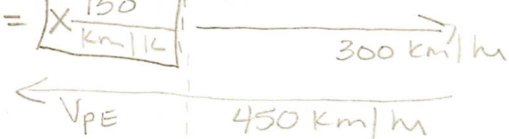
$\tan^{-1} = \frac{y}{x} = \frac{126}{40} =$

2) $V_{WE} = 300$ km/hr

$V_{PW} = 450$ km/hr.

$\Delta d_x = 250$ km

$V_{PE} =$ ~~x~~ 150 km/hr



$V = 150$ $V = \frac{d}{t}$

$d = 250$ $150 = \frac{250}{t}$

$t = x$

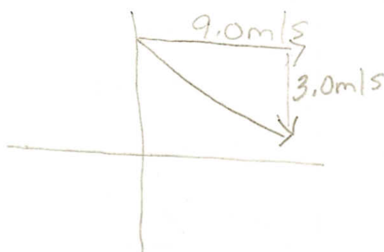
$\frac{250}{150} = t =$ 1.7 hr

3) $V_{TW} = 9.0$ m/s

$V_{WE} = 3.0$ m/s

$V_{TE} = ?$

$d = x$



$\tan^{-1} = \frac{3}{9} =$ 18° E of N

$V_{TE} = V_{TW} + V_{WE}$

$= \sqrt{9.0^2 + 3.0^2}$

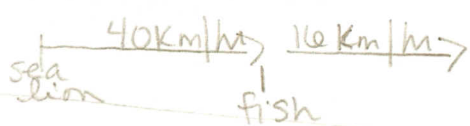
$= 9.49$ m/s x 60 s = 570 m

4) $V_{SW} = 40$ km/hr

$d = 60$ km

$V_{FW} = 16$ km/hr

$V_{SF} = x$



$V_{SF} = V_{SW} + {}^{-}V_{WF}$

$= 40 + {}^{-}16 =$

$= 24$ km/hr toward fish

$V = \frac{d}{t}$

$\frac{24 \text{ km}}{\text{hr}} = 6.67$ m/s

$\frac{60}{6.67} =$ 9 s

5) V_{1E} 90 km/hr → V_{2E} ← -90 km/hr

$d = x$

$t = 40$ s

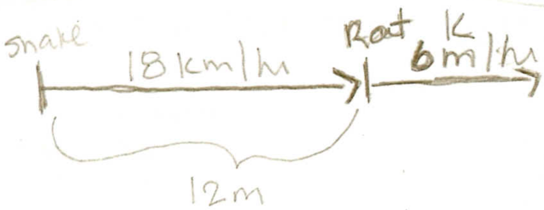
V_{12} $V_{1E} + V_{E2}$

$V_{12} = V_{1E} - V_{2E} = V_{1E} + {}^{+}V_{E2}$ (90 + +90)

$= 90 + 90 = 180$ km/hr. $= 50$ m/s

$V = \frac{d}{t} = 50 = \frac{x}{40} =$ 2000 m or 2 km

6)



$d = 12$ m

$V_{mE} = 18$ km/hr $= 5$ m/s

$V_{rE} = 6$ km/hr $= 1.67$ m/s

$V_{mr} = V_{mE}$

$V_{mr} = V_{mE} + V_{rE}$

$= V_{mE} + {}^{-}V_{Er}$

$= 18 + {}^{-}6 = 12$ km/hr. $= 3.33$ m/s

$V = \frac{d}{t}$ $3.33 = \frac{12}{x} =$ 3.6 s