Two-Dimensional Motion and Vectors

**Problem C**

**ADDING VECTORS ALGEBRAICALLY**

**PROBLEM**

The record for the longest nonstop closed-circuit flight by a model airplane was set in Italy in 1986. The plane flew a total distance of 1239 km. Assume that at some point the plane traveled $1.25 \times 10^3$ m to the east, then $1.25 \times 10^3$ m to the north, and finally $1.00 \times 10^3$ m to the southeast. Calculate the total displacement for this portion of the flight.

**SOLUTION**

1. **DEFINE**

   **Given:**
   
   \[ d_1 = 1.25 \times 10^3 \text{ m} \quad d_2 = 1.25 \times 10^3 \text{ m} \quad d_3 = 1.00 \times 10^3 \text{ m} \]

   **Unknown:**
   
   \[ \Delta x_{\text{tot}} = ? \quad \Delta y_{\text{tot}} = ? \quad d = ? \quad \theta = ? \]

2. **PLAN**

   **Choose the equation(s) or situation:** Orient the displacements with respect to the x-axis of the coordinate system.
   
   \[ \theta_1 = 0.00^\circ \quad \theta_2 = 90.0^\circ \quad \theta_3 = -45.0^\circ \]
   
   Use this information to calculate the components of the total displacement along the x-axis and the y-axis.
   
   \[ \Delta x_{\text{tot}} = \Delta x_1 + \Delta x_2 + \Delta x_3 \]
   
   \[ = d_1(\cos \theta_1) + d_2(\cos \theta_2) + d_3(\cos \theta_3) \]
   
   \[ \Delta y_{\text{tot}} = \Delta y_1 + \Delta y_2 + \Delta y_3 \]
   
   \[ = d_1(\sin \theta_1) + d_2(\sin \theta_2) + d_3(\sin \theta_3) \]

   Use the components of the total displacement, the Pythagorean theorem, and the tangent function to calculate the total displacement.
   
   \[ d = \sqrt{\left(\Delta x_{\text{tot}}\right)^2 + \left(\Delta y_{\text{tot}}\right)^2} \quad \theta = \tan^{-1}\left(\frac{\Delta y_{\text{tot}}}{\Delta x_{\text{tot}}}\right) \]

3. **CALCULATE**

   **Substitute the values into the equation(s) and solve:**
   
   \[ \Delta x_{\text{tot}} = (1.25 \times 10^3 \text{ m})(\cos 0^\circ) + (1.25 \times 10^3 \text{ m})(\cos 90.0^\circ) \]
   
   \[ + (1.00 \times 10^3 \text{ m})(\cos -45.0^\circ) \]
   
   \[ = 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m} \]
   
   \[ = 1.96 \times 10^3 \text{ m} \]
   
   \[ \Delta y_{\text{tot}} = (1.25 \times 10^3 \text{ m})(\sin 0^\circ) + (1.25 \times 10^3 \text{ m})(\sin 90.0^\circ) \]
   
   \[ + (1.00 \times 10^3 \text{ m})(\sin -45.0^\circ) \]
   
   \[ = 1.25 \times 10^3 \text{ m} + 7.07 \times 10^2 \text{ m} \]
   
   \[ = 0.543 \times 10^3 \text{ m} \]
   
   \[ d = \sqrt{(1.96 \times 10^3 \text{ m})^2 + (0.543 \times 10^3 \text{ m})^2} \]
\[ d = \sqrt{3.84 \times 10^6 \text{ m}^2 + 2.95 \times 10^5 \text{ m}^2} = \sqrt{4.14 \times 10^6 \text{ m}^2} \]

\[ d = 2.03 \times 10^3 \text{ m} \]

\[ \theta = \tan^{-1}\left(\frac{0.543 \times 10^3 \text{ m}}{1.96 \times 10^3 \text{ m}}\right) \]

\[ \theta = 15.5^\circ \text{ north of east} \]

**4. EVALUATE** The magnitude of the total displacement is slightly larger than that of the total displacement in the eastern direction alone.

**ADDITIONAL PRACTICE**

1. For six weeks in 1992, Akira Matsushima, from Japan, rode a unicycle more than 3000 mi across the United States. Suppose Matsushima is riding through a city. If he travels 250.0 m east on one street, then turns counterclockwise through a 120.0° angle and proceeds 125.0 m northwest along a diagonal street, what is his resultant displacement?

2. In 1976, the Lockheed SR-71A *Blackbird* set the record speed for any airplane: \(3.53 \times 10^3\) km/h. Suppose you observe this plane ascending at this speed. For 20.0 s, it flies at an angle of 15.0° above the horizontal, then for another 10.0 s its angle of ascent is increased to 35.0°. Calculate the plane's total gain in altitude, its total horizontal displacement, and its resultant displacement.

3. Magnor Mydland of Norway constructed a motorcycle with a wheelbase of about 12 cm. The tiny vehicle could be ridden at a maximum speed 11.6 km/h. Suppose this motorcycle travels in the directions \(d_1\) and \(d_2\), where \(d_1\) is 30° with the horizontal (upward and right) and \(d_2\) is 45° with the vertical (down and to the right). The net vertical displacement is zero. Calculate \(d_1\) and \(d_2\) and determine how long it takes the motorcycle to reach a net displacement of 2.0 \(\times\) 10^2 m to the right.

4. The fastest propeller-driven aircraft is the Russian TU-95/142, which can reach a maximum speed of 925 km/h. For this speed, calculate the plane's resultant displacement if it travels east for 1.50 h, then turns 135° northwest and travels for 2.00 h.

5. In 1952, the ocean liner *United States* crossed the Atlantic Ocean in less than four days, setting the world record for commercial ocean-going vessels. The average speed for the trip was 57.2 km/h. Suppose the ship moves in a straight line eastward at this speed for 2.50 h. Then, due to a strong local current, the ship's course begins to deviate northward by 30.0°, and the ship follows the new course at the same speed for another 1.50 h. Find the resultant displacement for the 4.00 h period.
1)\[ x = 250 \text{ m}, \quad y = 0 \]

2)\[ x_{\text{tot}} = 187.5 \]
\[ y_{\text{tot}} = 108 \]

\[ \theta = \frac{187.5}{108} \]

\[ 187.5^2 + 108^2 = R^2 \]
\[ 31564.41 + 11664 = R^2 \]
\[ R = 29019.40 \]

2)\[ \frac{3530 \text{ km}}{\text{s}} \times \frac{1 \text{ km}}{3600 \text{ s}} \times 1000 \text{ m} = 980.56 \text{ m/s} \]

2)\[ 980.56 \times 20 \text{ s} = 19611.2 \]

\[ x = \cos(15)(19611) = 18942.96 \]
\[ \sin(15)(19611) = 5075.7 \]

\[ x_{\text{tot}} = 18942.96 + 8031.78 = 26974.74 \]
\[ y_{\text{tot}} = 5624.26 + 5075.7 = 10699.96 \]

\[ \theta = \tan^{-1} \left( \frac{10699.96}{26974.74} \right) = 21.6^\circ \text{ N} 80^\circ \text{ E} \]
4)  
1) \[ X = 1387.50 \]
   \[ q = 0 \]
   \[ 1387.50 \]
   \[ 925 \times 1.5 = \]

   \[ R = \sqrt{79.5^2 + 1308^2} \]
   \[ R = 1310 \]
   \[ +90 = 308 \]
   \[ \theta = \frac{79.5}{1308} \]
   \[ \theta = 3.5 \]
   \[ EQN \]

2) \[ x = \cos 45(1850) = -1308 \]
   \[ y = \sin 45(1850) = 1308 \]

   \[ x_{tot} = 1387.50 + (-1308) = 79.5 \]
   \[ y_{tot} = 1308 + 0 = 1308 \]

5)  
1) \[ X = 143 \]
   \[ Y = 0 \]

   \[ 57.2 \times 2.50 = \]

   \[ x_{tot} = 74.3 + 143 = 217.3 \]
   \[ y_{tot} = 0 + 42.9 = 42.9 \]

   \[ R = 217.3^2 + 42.9^2 = 221.49 \]

   \[ \theta = \frac{42.9}{217.3} (\tan^{-1}) = 11.2 \]
   \[ N6\theta E. \]
1) $d_1 = \frac{d_1 y}{\sin \theta_1 (d_1)}$

2) $d_1 y = -d_2 y$

\[ R^2 = x_{tot}^2 + y_{tot}^2 \]

\[ d_1 \frac{\sin \theta_2}{\sin \theta_1} = -d_2 \frac{\sin \theta_2}{\sin \theta_1} \]

\[ d_1 = -d_2 \frac{\sin \theta_2}{\sin \theta_1} \]

\[ d_1 = -d_2 \left( \frac{\sin -45}{\sin 30} \right) \]

\[ d_1 = -d_2 \left( -\frac{\sqrt{2}}{2} \right) \]

\[ d_1 = -d_2 \left( \frac{1.707}{0.5} \right) \]

\[ d_1 = -d_2 (1.414) \]

\[ x_{tot} = d_1 x \cos \theta_1 + d_2 x \cos \theta_2 \]

\[ x_{tot} = d_2 (1.414) \cos 30 + d_2 x \cos 45 \]

\[ X_{tot} = d_2 (1.22) + d_2 (0.707) \]

\[ 200 = d_2 (1.22) + d_2 (0.707) \]

\[ 200 = 1.93 d_2 \]

\[ d_2 = \frac{200}{1.93} = 103.4 \]

\[ d_1 = d_2 (1.414) = \frac{d_1}{d_2} \]

\[ = 103.4 (1.414) = 146.49 \]